Exercise Sheet 7 — Discrete Probability

Course: STAT 150

Due: October 16, 2026

Solutions to every exercise are typeset here too, hidden behind `print=false` by default, and collected into an answer key at the end with `\printsolutions` — so the same source file produces both the student handout and the instructor key.

Exercise 1

_____/4 p.

A fair six-sided die is rolled twice. What is the probability that the sum of the two rolls is 7?

Exercise 2 *Bayes' rule*

_____/6 p.

A test for a disease is 99% accurate in both directions. The disease affects 1% of the population. Given a positive test result, what is the probability the patient actually has the disease?

Exercise 3

_____/ 8 (+2) p.

Let X and Y be independent Poisson random variables with parameters λ and μ. Show that $X + Y$ is Poisson with parameter $\lambda + \mu$.

Answer Key

Solutions to the Exercises

Solution 1

4 p.

There are 36 equally likely outcomes. The pairs summing to 7 are $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$ — six of them. So the probability is $6/36 = 1/6$.

Solution 2

6 p.

Let D be "has disease" and $+$ be "tests positive". By Bayes' rule,

$$\Pr(D \mid +) = \frac{\Pr(+ \mid D)\Pr(D)}{\Pr(+ \mid D)\Pr(D) + \Pr(+ \mid \neg D)\Pr(\neg D)} = \frac{0.99 \times 0.01}{0.99 \times 0.01 + 0.01 \times 0.99} = 0.5.$$

So despite the test's accuracy, only about half of positive results are true positives, because the disease is rare.

Solution 3

8 (+2) p.

For $n \geq 0$,

$$\Pr(X + Y = n) = \sum_{k=0}^{n} \Pr(X = k)\Pr(Y = n - k) = \sum_{k=0}^{n} e^{-\lambda}\frac{\lambda^k}{k!}e^{-\mu}\frac{\mu^{n-k}}{(n-k)!}.$$

Factoring out $e^{-(\lambda+\mu)}/n!$ and recognising the remaining sum as the binomial expansion of $(\lambda+\mu)^n$ via the binomial theorem gives

$$\Pr(X + Y = n) = e^{-(\lambda+\mu)}\frac{(\lambda+\mu)^n}{n!},$$

which is exactly the Poisson$(\lambda + \mu)$ probability mass function. *(Bonus: this is a special case of the general fact that Poisson random variables are closed under convolution.)*