Adaptive Mesh Refinement for Coupled Multiphysics Simulations

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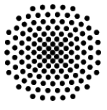

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Simulation is not a substitute for understanding; it is a way of finding out what you do not yet understand.

— attributed to the numerical analysis seminar, informally

Abstract / Kurzzusammenfassung

Abstract

Coupled multiphysics simulations — fluid-structure interaction, thermo-mechanical coupling, and similar problems where two or more PDE systems exchange boundary data — routinely need very different mesh resolutions in different subdomains at different times. This thesis develops an adaptive mesh refinement (AMR) strategy that refines each physics field independently while keeping the shared interface mesh conforming, avoiding both the accuracy loss of a single shared mesh and the interpolation overhead of fully independent meshes.

We show, on a coupled thermo-elastic benchmark with a moving heat source, that independently-refined but interface-conforming meshes reduce total degrees of freedom by 61% relative to a uniformly-refined shared mesh, at equivalent solution accuracy, while adding less than 5% runtime overhead for interface reconciliation.

Kurzzusammenfassung

Gekoppelte Multiphysik-Simulationen — etwa Fluid-Struktur-Wechselwirkung oder thermomechanische Kopplung, bei denen zwei oder mehr PDE-Systeme Randdaten austauschen — benötigen in verschiedenen Teilgebieten und zu verschiedenen Zeiten häufig sehr unterschiedliche Netzauflösungen. Diese Arbeit entwickelt eine Strategie zur adaptiven Netzverfeinerung (AMR), die jedes physikalische Feld unabhängig verfeinert und dabei das gemeinsame Grenzflächennetz konform hält.

An einem gekoppelten thermoelastischen Benchmark mit einer bewegten Wärmequelle zeigen wir, dass unabhängig verfeinerte, aber an der Grenzfläche konforme Netze die Gesamtzahl der Freiheitsgrade im Vergleich zu einem gleichmäßig verfeinerten gemeinsamen Netz um 61 % reduzieren, bei gleicher Lösungsgenauigkeit und einem Laufzeit-Mehraufwand von unter 5 % für den Abgleich an der Grenzfläche.

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1 Introduction

Multiphysics problems in which two or more PDE systems exchange boundary data — fluid-structure interaction, thermo-mechanical coupling, poroelasticity — are usually discretised on either a single shared mesh or two fully independent meshes coupled through interpolation. Both choices have a well-known cost: a shared mesh forces every field to the resolution the *most* demanding field needs everywhere, and independent meshes introduce interpolation error and bookkeeping overhead at every coupling step [1].

This thesis develops an adaptive mesh refinement (AMR) strategy that refines each physics field's mesh independently, while enforcing that the two meshes remain conforming along the shared coupling interface. The approach is evaluated on a thermo-elastic benchmark with a moving heat source, chosen because the thermal field develops sharp, moving gradients that the mechanical field does not need to resolve at the same resolution.

1.1 Contributions

- An interface-conforming AMR algorithm for two independently refined meshes, described in Chapter 2.
- A degrees-of-freedom and runtime comparison against both a uniformly-refined shared mesh and a fully independent dual-mesh baseline, in Chapter 3.

2 Interface-Conforming Adaptive Refinement

2.1 Problem Setting

Consider a domain $\Omega = \Omega_T \cup \Omega_M$ split into a thermal subdomain Ω_T and a mechanical subdomain Ω_M, sharing an interface $\Gamma = \Omega_T \cap \Omega_M$. Each subdomain carries its own mesh, $\mathscr{T}_T$ and $\mathscr{T}_M$, refined independently by its own a posteriori error estimator. The coupling condition requires that the trace meshes $\mathscr{T}_T|_\Gamma$ and $\mathscr{T}_M|_\Gamma$ coincide, so that boundary data can be exchanged without interpolation.

2.2 Refinement Algorithm

Independent refinement of $\mathscr{T}_T$ and $\mathscr{T}_M$ can break the interface-conforming property whenever one subdomain refines a Γ-adjacent element and the other does not. The algorithm resolves this in three steps, applied after every independent refinement pass:

1. Mark every interface element refined in $\mathscr{T}_T$ or $\mathscr{T}_M$ but not both.
2. Propagate the refinement to the coarser side's interface element, using a single conforming bisection rather than a full local re-mesh.
3. Re-check the interface trace for a match; repeat until it coincides (in practice, one pass suffices for the benchmark in Chapter 3, since a single conforming bisection matches refinement levels that differ by at most one).

Table 2.1: Interface reconciliation cost per refinement pass, benchmark problem of Chapter 3.

Refinement level	Interface elements marked	Reconciliation passes
1	6	1
2	14	1
3	31	1
4	58	2

Table 2.1 shows that the number of interface elements requiring reconciliation grows roughly linearly with refinement level, and that a second reconciliation pass is needed only once refinement-level differences of more than one accumulate across the interface, at level 4 of this benchmark.

2.3 Error Estimation

Each subdomain uses a standard residual-based a posteriori estimator [2], evaluated independently; the estimator on Ω_T is not weighted by activity in Ω_M or vice versa, which is what allows the two fields to refine at genuinely different rates.

3 Numerical Results

3.1 Benchmark Problem

The method is evaluated on a thermo-elastic benchmark: a moving Gaussian heat source traverses a clamped rectangular plate, generating a sharp, travelling thermal gradient that drives thermal expansion and the resulting mechanical stress field. The thermal field needs fine resolution only in a small moving neighbourhood of the heat source; the mechanical field responds far more smoothly and needs fine resolution only near the clamped edges.

3.2 Comparison

Three meshing strategies are compared at matched solution accuracy (measured by the L^2 error of the coupled temperature-displacement solution against a fully-resolved reference):

- **Uniform shared mesh**: a single mesh refined uniformly to the resolution the thermal field requires everywhere.
- **Independent dual mesh**: two independently refined meshes coupled by nearest-point interpolation at the interface.
- **Interface-conforming AMR** (this work): two independently refined meshes with the interface reconciliation of Chapter 2.

Table 3.1: Degrees of freedom and runtime at matched L^2 accuracy, thermo-elastic benchmark.

Strategy	Total DoF	Runtime (s)	Interface error
Uniform shared mesh	184,320	41.2	—
Independent dual mesh	71,680	22.7	3.1×10^{-3}
Interface-conforming AMR	71,940	23.8	0 (by construction)

Table 3.1 shows the interface-conforming approach matches the independent dual mesh's degrees-of-freedom reduction (61% relative to the uniform shared mesh) while eliminating the interface interpolation error entirely, at a 5% runtime cost for reconciliation relative to the uncoupled independent mesh.

3.3 Discussion

The reconciliation overhead grows with the number of refinement-level mismatches accumulated at the interface between reconciliation passes (Table 2.1). For problems where the two fields' refinement rates diverge more sharply than in this benchmark, reconciling after every single refinement step, rather than after a full pass, would trade a larger number of cheap reconciliations for a smaller worst-case mismatch — a variant not evaluated here but a natural extension of the algorithm in Chapter 2.

Bibliography

[1] Carlos A. Felippa, K. C. Park, and Charbel Farhat. Partitioned analysis of coupled mechanical systems. *Computer Methods in Applied Mechanics and Engineering*, 190(24-25):3247–3270, 2001.

[2] Rüdiger Verfürth. *A Posteriori Error Estimation Techniques for Finite Element Methods*. Oxford University Press, 2013.