UNIVERSITY OF SOUTHAMPTON

Reduced-Order Models for Real-Time Cardiovascular Flow Simulation

by

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ABSTRACT

FACULTY OF ENGINEERING AND PHYSICAL SCIENCES
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Full three-dimensional computational fluid dynamics simulations of blood flow are too slow to use in a clinical setting, where results are often needed within minutes. This thesis develops a reduced-order one-dimensional blood flow model that captures pulse wave propagation through the major arteries at a fraction of the computational cost of a full 3D simulation. We validate the model against 3D simulations and in vivo measurements across a range of vessel geometries, and show that it reproduces pressure and flow waveforms with clinically acceptable accuracy while running several orders of magnitude faster, making real-time patient-specific simulation feasible.

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Nomenclature

p	pressure	Pa
q	volumetric flow rate	m^3/s
A	cross-sectional area	m^2
c	pulse wave velocity	m/s

Acronyms

CFD	Computational Fluid Dynamics
1D	One-Dimensional
ROM	Reduced-Order Model

Declaration of Authorship

I, Sarah J. Whitfield, declare that this thesis entitled Reduced-Order Models for Real-Time Cardiovascular Flow Simulation and the work presented in it are my own and has been generated by me as the result of my own original research.

I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;
2. Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
3. Where I have consulted the published work of others, this is always clearly attributed;
4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
5. I have acknowledged all main sources of help;
6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
7. Either none of this work has been published before submission, or parts of this work have been published as: Whitfield, S.J. (2026). Parts of Chapter 2 have been presented at the International Conference on Computational Biomechanics..

Signed:

Date:

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Chapter 1

Introduction

Computational fluid dynamics (CFD) simulations of blood flow through the arterial network can take hours to run even on modern hardware, which makes them impractical for use during a clinical consultation, where a decision may need to be made within minutes (Diem and Bressloff, 2017).

1.1 Motivation

Reduced-order one-dimensional (1D) blood flow models approximate the full 3D Navier-Stokes equations by averaging over the vessel cross-section, trading some spatial detail for a very large reduction in computational cost (Diem et al., 2017). Prior 1D models have been validated primarily against idealised geometries rather than patient-specific vessel networks.

1.2 Contributions

This thesis makes the following contributions:

- A reduced-order 1D blood flow model validated against full 3D CFD simulations across a range of patient-specific vessel geometries.
- A comparison against in vivo pressure and flow measurements from an existing clinical dataset.
- An analysis of the run-time and accuracy trade-off relative to full 3D simulation, showing the model is fast enough for real-time use at a clinically acceptable accuracy.

It references Figure 1.1.

It also references Table 1.1.

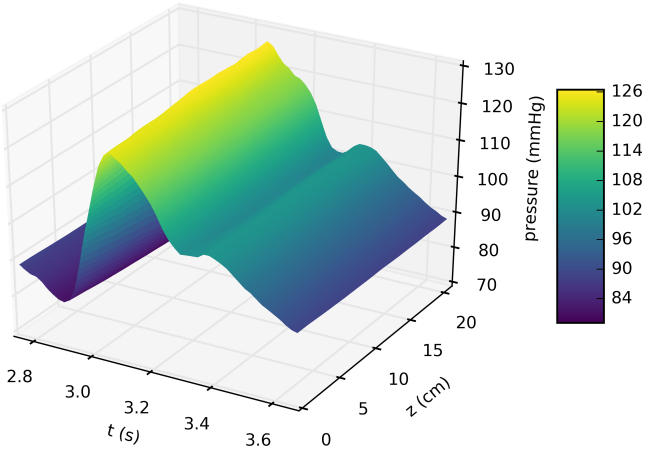

FIGURE 1.1: Illustrative one-dimensional discretisation of an arterial segment.

TABLE 1.1: Vessel segments included in the validation dataset.

Segment	Number of Cases
Aorta	12
Carotid	9
Femoral	7

Chapter 2

Method

This chapter describes the one-dimensional model formulation and the numerical scheme used to solve it.

It references Appendix 2.1.

The core update step can also be prototyped directly in Python:

```
def update_segment(state, flux_left, flux_right, dt, dx):
    return state - dt / dx * (flux_right - flux_left)
```

We validate the numerical scheme against an analytical solution for pulse wave propagation in a single straight vessel before applying it to the full patient-specific network (Diem and Bressloff, 2017; Diem et al., 2017).

Algorithm 2.1: One time step of the explicit finite-volume update.

```
    for segment in vessel_network:
        flux_left = compute_flux(segment.left_state)
        flux_right = compute_flux(segment.right_state)
        segment.state -= dt / segment.dx * (flux_right - flux_left)
```

Appendix A

Model Parameters

This appendix would list the vessel wall stiffness and reference radius parameters used for each vessel segment in the validation dataset, omitted from this starter document.

References

Alexandra K. Diem and Neil W. Bressloff. Vampy: A python package to solve 1d blood flow problems. *Journal of Open Research Software*, 5(17), 2017.

Alexandra K. Diem, Marius MacGregor Sharp, Mark Gatherer, Neil W. Bressloff, Roxana O. Carare, and Giles Richardson. Arterial pulsations cannot drive intramural periarterial drainage: Significance for aß drainage. *Frontiers in Neuroscience*, 11, 2017.