STAT 210

Problem Set 4: Discrete Random Variables

Term: Fall 2026 |

Instructions

Complete every problem below. Show your working; a bare final answer earns partial credit at most.

[EXR] Exercise 1 (Expectation)
A fair die is rolled once. Let X be the value shown. Compute $\mathbb{E}[X]$ and $\mathrm{Var}(X)$.

Solution:

$\mathbb{E}[X] = \frac{1}{6}\sum_{k=1}^{6} k = \frac{21}{6} = 3.5$.
$\mathbb{E}[X^2] = \frac{1}{6}\sum_{k=1}^{6} k^2 = \frac{91}{6}$, so $\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = \frac{91}{6} - \frac{49}{4} = \frac{35}{12}$.

[EXR] Exercise 2 (Binomial distribution)
A biased coin lands heads with probability $p = 0.3$. It is flipped 10 times. What is the probability of exactly 3 heads?

Solution:

Let $X \sim \mathrm{Binomial}(10, 0.3)$. Then

$$\Pr(X = 3) = \binom{10}{3}(0.3)^3(0.7)^7 \approx 0.2668.$$

[EXR] Exercise 3 (Independence)
Suppose X and Y are independent random variables with $\mathrm{Var}(X) = 4$ and $\mathrm{Var}(Y) = 9$. Find $\mathrm{Var}(2X - Y)$.

Solution:

Since X and Y are independent, cross terms vanish:

$$\mathrm{Var}(2X - Y) = 4\,\mathrm{Var}(X) + \mathrm{Var}(Y) = 4(4) + 9 = 25.$$

[EXR] Exercise 4 (Proof)

Prove that for any random variable X with finite variance, $\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$.

Proof: By definition, $\mathrm{Var}(X) = \mathbb{E}\big[(X - \mathbb{E}[X])^2\big]$. Expanding the square,

$$\mathbb{E}\big[(X - \mathbb{E}[X])^2\big] = \mathbb{E}\big[X^2 - 2X\mathbb{E}[X] + \mathbb{E}[X]^2\big] = \mathbb{E}[X^2] - 2\mathbb{E}[X]^2 + \mathbb{E}[X]^2 = \mathbb{E}[X^2] - \mathbb{E}[X]^2,$$

using linearity of expectation and that $\mathbb{E}[X]$ is a constant. ■