Real Analysis – Assignment 6

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Problem 1

Let (a_n) be a sequence of real numbers. Prove that if (a_n) converges, then (a_n) is bounded.

Proof. Suppose $a_n \to L$. Taking $\varepsilon = 1$ in the definition of convergence, there exists N such that $|a_n - L| < 1$ for all $n > N$, so $|a_n| < |L| + 1$ for all $n > N$. Let

$$M = \max\{|a_1|, |a_2|, \ldots, |a_N|, |L| + 1\}.$$

Then $|a_n| \leq M$ for every n, so (a_n) is bounded. □

Problem 2

Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges, and if so, whether it converges absolutely.

Claim. The series converges conditionally but not absolutely.

Proof. The terms $1/\sqrt{n}$ decrease to 0, so the alternating series test gives convergence of $\sum(-1)^n/\sqrt{n}$. However, $\sum|(-1)^n/\sqrt{n}| = \sum 1/\sqrt{n}$ is a p-series with $p = 1/2 \leq 1$, which diverges. Hence the original series converges conditionally but not absolutely. □