Foundations of Analytic Structures

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Preface

This book grew out of a lecture series on analytic structures given at the Institute of Analytical Engines. It is intended as a graduate-level introduction, assuming only a first course in real analysis.

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CHAPTER 1

Preliminaries

We fix notation and recall the basic definitions used throughout.

Definition 1.1 (Analytic structure). An *analytic structure* on a set X is a collection $\mathcal{A}$ of functions $X \to \mathbb{C}$ closed under addition, multiplication, and locally uniform limits.

Theorem 1.2. *Every analytic structure $\mathcal{A}$ on a compact set X contains the constant functions.*

Proof. Let $c \in \mathbb{C}$ be constant. The constant function c is the locally uniform limit of the sequence $c, c, c, \dots \in \mathcal{A}$, hence lies in $\mathcal{A}$ by closure under limits. $\square$

Remark 1.3. Theorem 1.2 is the reason we always assume $X \neq \emptyset$.

See [**2**] for the classical real-analysis background, and [**1**] for the complex-analytic prerequisites.

CHAPTER 2

Convergence and Compactness

PROPOSITION 2.1. *If X is compact and $\mathcal{A}$ separates points, then the evaluation map $X \to \mathbb{C}^{\mathcal{A}}$ is an embedding.*

PROOF. Continuity and injectivity follow from separation of points (Definition 1.1); compactness of X upgrades the continuous injection to a homeomorphism onto its image, by the standard closed-map lemma. □

COROLLARY 2.2. *Under the hypotheses of Proposition 2.1, X is metrizable.*

Bibliography

1. Lars V. Ahlfors, *Complex analysis*, 3 ed., McGraw-Hill, 1979.
2. Walter Rudin, *Principles of mathematical analysis*, 3 ed., McGraw-Hill, 1976.